From the similar triangles PAB and PBD, we have $PB:DB=PA:AB$, or $PB:DB=h:a$. But $DB=\sqrt{(AD^2-AB^2)}=\sqrt{(4R^2-a^2)}$.

$\therefore$ $PB:\sqrt{(4R^2-a^2)}=h:a$. Whence $PB=\dfrac{h\sqrt{(4R^2-a^2)}}{a}$.

$$\therefore \frac{h^2\,(4R^2-a^2)}{a^2}=h(h+2R).$$

$$\therefore h=\frac{2Ra^2}{4R^2-2a^2}=\frac{Ra^2}{2R^2-a^2}=5.0062 \text{ miles}.$$

Also solved by *G. B. M. ZERR*, *L. C. WALKER*, *H. C. WHITAKER*, and *J. SCHEFFER*.

CALCULUS.

127. Proposed by J. A. CALDERHEAD, B.Sc., Professor of Mathematics, Curry University, Pittsburg. Pa.

Find the moment of inertia of a parallelogram about an axis perpendicular to its plane and passing through the intersection of its diagonals.

Solution by G. B. M. ZERR, A.M.. Ph. D., The Temple College. Philadelphia, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and J. M. ARNOLD, Crompton, R. I.

Let the intersection of the diagonals be the origin; straight lines through the origin parallel to the sides, the axes; a, b the sides.

Then the required moment of inertia is

$$I=\rho\sin\beta\int_{-\frac{1}{2}a}^{\frac{1}{2}a}\int_{-\frac{1}{2}b}^{\frac{1}{2}b}[(x+y\cos\beta)^2+y^2\sin^2\beta]dxdy$$

where β is the acute angle of the parallelogram.

$$\therefore I=\rho\sin\beta\int_{-\frac{1}{2}a}^{\frac{1}{2}a}(bx^2+\tfrac{1}{12}b^3)dx=\tfrac{1}{12}\rho ab\sin\beta(a^2+b^2)=\tfrac{1}{12}m(a^2+b^2).$$

Also solved by *W. J. GREENSTREET* and *L. C. WALKER*.

128. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The differential equation of a curve is $\dfrac{d^3y}{dx^3}+y=0$. Find its equation, there being the additional conditions that for $x=0$, $y=1$, that the tangent at the point (0, 1) makes an angle of 45° with the axes, and finally that that point is a point of inflexion.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.; and L. C. WALKER, A. M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Using D for the operator d/dx, the given equation is

$$(D^3+1)y=0\ldots\ldots(1),\text{ or }(D^2-D+1)(D+1)y=0\ldots\ldots(2).$$

Integrating $(D+1)y=e^{\frac{1}{2}x}(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x)$......(3).

Differentiating this with respect to x,

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{\frac{1}{2}x}[(\tfrac{1}{2}\cos\frac{\sqrt{3}}{2}x-\frac{\sqrt{3}}{2}\sin\frac{\sqrt{3}}{2}x)A$$

$$+(\tfrac{1}{2}\sin\frac{\sqrt{3}}{2}x+\frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}x)B]......(4).$$

When $x=0$, $y=1$, $dy/dx=1$, and (3) gives $A=2$.
When $x=0$, $D=1$, $D^2=0$, and (4) gives $B=0$.
$\therefore$ (3) is $(D+1)y=2e^{\frac{1}{2}x}\cos\frac{\sqrt{3}}{2}x$......(5).

Integrating (5), and noticing that when $x=0$, and $y=1$, the constant of integration is nought,

$$y=e^{\frac{1}{2}x}(\tfrac{1}{3}\sqrt{3}\sin\frac{\sqrt{3}}{2}x+\cos\frac{\sqrt{3}}{2}x)......(6).$$

Also solved by *G. B. M. ZERR* and *J. SCHEFFER*.

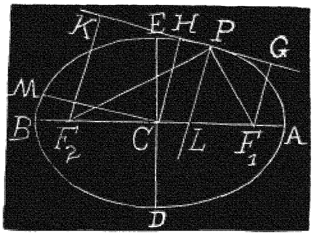
MECHANICS.

128. Proposed by M. E. GRABER, A. B., Heidelberg University, Tiffin, Ohio.

A particle is placed on the convex side of a smooth ellipse and is acted upon by two forces, F and F', towards the foci, and a force, F'', towards the center. Find the position of equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and the PROPOSER.

Let C be the center of the ellipse, semi-axes a, b; F_1, F_2, the foci; force $PF_1=F$, $PF_2=F'$, $PC=F''$; distance $PC=r$. Let fall the perpendiculars F_1G, CH, F_2K from the foci F_1, F_2, and the center C on the tangent at P.



Let $\angle PF_1G=\angle PF_2K=\theta$, $\angle PCH=\phi$.
Then $F\sin\theta=F'\sin\theta+F''\sin\phi$.

$\therefore \dfrac{\sin\phi}{\sin\theta}=\dfrac{F-F'}{F''}=n$, suppose.

$F_2K=F_2P\cos\theta$, $F_1G=F_1P\cos\theta$, $CH=r\cos\phi$.
$2CH=F_2K+F_1G=(F_2P+F_1P)\cos\theta=2a\cos\theta$.
$\therefore r\cos\phi=a\cos\theta$.
$F_2K.F_1G=b^2=F_2P.F_1P\cos^2\theta=CM^2\cos^2\theta=(a^2+b^2-r^2)\cos^2\theta$.

$$\therefore \cos\theta=\frac{b}{\sqrt{(a^2+b^2-r^2)}},\quad \sin\theta=\frac{\sqrt{(a^2-r^2)}}{\sqrt{(a^2+b^2-r^2)}}.$$